On the Definition of Gauge Field Operators in Lattice Gauge-Fixed Theories

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We address the problem of defining the gluon field on the lattice in terms of the natural link variables. Different regularized definitions are shown, through non perturbative numerical computation, to converge towards the same continuum renormalized limit.

This talk, based on ref. [1] to which we refer for details, is divided in two parts. In the first one we will discuss the definition of the gauge potential $A_\mu(x)$ on the lattice and we will show that different definitions could have strong effects on gauge dependent quantities which are relevant in the gauge-fixing procedure. Nevertheless we will show in the second part that different definitions of the gluon field on the lattice give rise to Green's functions proportional to each other at the non-perturbative level. This important feature is necessary in order to garantee the uniqueness of the renormalized continuum operators.

The usual definition of the 4-potential in terms of the links, U_μ, which represent the fundamental dynamical gluon variables, is given by

$$A_\mu(x) \equiv \frac{((U_\mu(x)) - (U^\dagger_\mu(x)))_{traceless}}{2iag_0}. \quad (1)$$

This definition is obtained taking an expansion in powers of a of the relation $U_\mu(x) \equiv \exp(ig_0 a A_\mu(x))$ that is naively suggested by the interpretation of $U_\mu(x)$ as the lattice parallel transport operator and by its formal expression in terms of the "continuum" gauge field variables, $A_\mu(x)$. The definition given in eq.(1) is not unique: it cannot be preferred to any other definition with analogue properties as, for instance:

$$A'_\mu(x) \equiv \frac{((U_\mu(x))^2 - (U^\dagger_\mu(x))^2)_{traceless}}{4iag_0}, \quad (2)$$

which in fact differs from eq.(1) by terms of $O(a^2)$ that formally go to zero as $a \to 0$.

From the algorithmical point of view, however, the various definitions are not interchangeable. In order to show this fact, let us suppose to fix the Landau gauge $\partial_\mu A_\mu$. First of all one has to choose the functional form of A_μ on the lattice in terms of the links and we adopt the usual definition eq.(1). Then the gauge is fixed applying a chain of gauge transformations $U^\Omega_\mu(x) \equiv \Omega(x)U_\mu(x)\Omega(x+\mu)^\dagger$ to a thermalized configuration until the control quantity $\theta^\Omega \sim \int d^4x(\partial_\mu A^\Omega_\mu)^2$ becomes very small, for example $\theta < 10^{-14}$. Let us now define θ' with the same functional form of θ but with A_μ replaced by A'_μ. The values of θ and θ', are shown in Fig.1 for a typical thermalized configuration, as functions of the lattice sweeps of the numerical gauge-fixing algorithm. As clearly seen θ' does not follow the same decreasing behaviour as θ: after an initial decrease, θ' goes to a constant value, many orders of magnitude higher than the corresponding value of θ. The marked difference between the two behaviours, already stressed in ref.[2], seems to cast doubts on the lattice gauge-fixing procedure and

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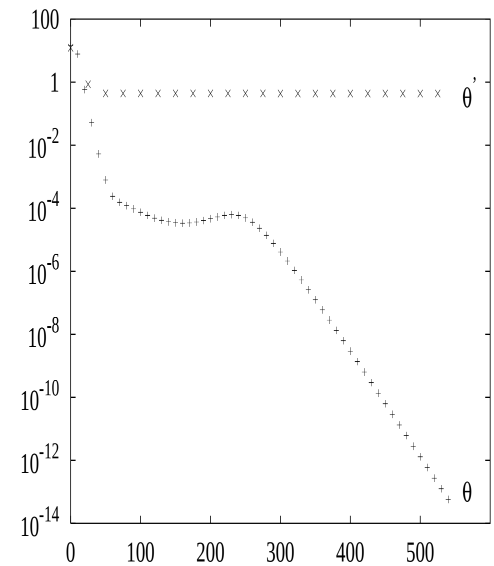

Figure 1. Typical behaviour of θ and $\theta^{'}$ vs gauge fixing sweeps at $\beta = 6.0$ for a thermalized $SU(3)$ configuration $8^3 \cdot 16$.

on the correspondence among the continuum limits of gauge-dependent operators having the same quantum numbers. This discrepancy vanishes if the comparison between two composite operators constructed in terms of A and $A^{'}$ is done by checking the values of the corresponding matrix elements as must be done in field theory. Then on the lattice one has to compare the average of the corresponding matrix elements taken on an ensemble of configurations. The relation between the two lattice definitions $A_\mu(x)$ and $A^{'}_\mu(x)$ is of the form: $A^{'}{}_\mu(x) = A_\mu(x) + a^2 W_\mu(x)$ where the $W_\mu(x)$ is a dimension 3 unrenormalized operator with the same quantum numbers of $A_\mu(x)$. The contribution of the operator W to the relation between A and $A^{'}$ in the continuum limit can not be neglected as it is shown from the following contruction of the renormalized operator: $W^R_\mu(x) = Z_W(g_0, a\,\mu_R)(W_\mu(x) + \frac{1-C(g_0)}{a^2} A_\mu(x))$ (C, as a consequence of the Callan-Symanzik equation, can only depend on the bare coupling g_0[5]) so that from the two last eqs. one obtains, up to terms truly of order a^2, $A^{'}{}_\mu(x) = C(g_0)A_\mu(x)$. This operatorial relation implies on Green's functions that we in general:

$$\frac{\langle \ldots A'_\mu(x) \ldots \rangle}{\langle \ldots A_\mu(x) \ldots \rangle} = C(g_0) \qquad (3)$$

We have numerically checked eq.(3) by measuring on different $SU(3)$ lattices, (see Table 1), in the Landau gauge with periodic boundary conditions, a few interesting correlators which are relevant to the investigation of the QCD gluon sector. The Landau gauge has been fixed in the standard way [3,4] minimizing, for each thermalized configuration, the usual functional F: $F[U^\Omega] \equiv -\frac{1}{V \cdot T} Re\ Tr \sum_\mu \sum_x U^\Omega_\mu(x)$.

In the following we will discuss these correlators:

$$\langle \mathcal{A}_0 \mathcal{A}_0 \rangle(t) \equiv \frac{1}{V^2} \sum_{\mathbf{x},\mathbf{y}} Tr\langle A_0(\mathbf{x},t) A_0(\mathbf{y},0)\rangle \qquad (4)$$

$$\langle \mathcal{A}_i \mathcal{A}_i \rangle(t) \equiv \frac{1}{3V^2} \sum_i \sum_{\mathbf{x},\mathbf{y}} Tr\langle A_i(\mathbf{x},t) A_i(\mathbf{y},0)\rangle \qquad (5)$$

using both A and $A^{'}$ as defined in eqs.(1), (2). Here and in the following we define $\mathcal{A}_\mu(t) = \sum_{\mathbf{x}} A_\mu(\mathbf{x},t)$. The correlation function $\langle \mathcal{A}_0 \mathcal{A}_0 \rangle(t)$, when evaluated through $A_\mu(x)$, is constant in t configuration by configuration, in virtue of the Landau gauge condition which, together with periodic boundary conditions, implies $\partial_0 \mathcal{A}_0 = 0$. The same should be true, on average, when $A^{'}$ is used. The behaviours of these correlators (that we do not show here) are well confirmed by our numerical simulations. It is surprising that also $\langle \mathcal{A}^{'}_0 \mathcal{A}^{'}_0 \rangle$ turns out to be constant configuration by configuration at the level of $\sim 5\%$, also because in this case the value of $\theta^{'}$ is different from zero on individual configurations as shown in Fig. 1.

In Fig. 2 the Green function $\langle \mathcal{A}^{'}_i \mathcal{A}^{'}_i \rangle$ and the rescaled one $C_i^2(g_0)\langle \mathcal{A}_i \mathcal{A}_i \rangle$ are reported for the run W60b. The remarkable agreement between these two quantities confirms the proportionality shown in eq. (3) (a triumph of field theory).

We have found that the proportionality factor, $C(g_0)$, may depend on the space-time direction

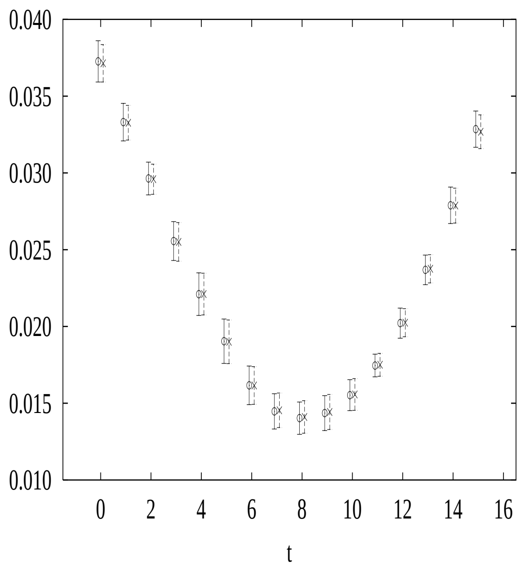

Figure 2. Comparison of the matrix elements of $\langle \mathcal{A}_i^{'} \mathcal{A}_i^{'} \rangle(t)$ (crosses) and the rescaled $\langle \mathcal{A}_i \mathcal{A}_i \rangle \cdot C_i^2(g_0)$ (open circles) as function of time for a set of 50 thermalized $SU(3)$ configurations at $\beta = 6.0$ with a volume $V \cdot T = 8^3 \cdot 16$ (run W60b). The data have been slightly displaced in t to help eye, the errors are jacknife.

μ. This fact is due to the breaking of cubic symmetry and it could be a potential source of systematic error in the non-perturbative evaluation of renormalization constants on asymmetric lattices. In our simulations, as shown in Table 1, two of the lattices (W60b, W64), have the time extension different from the spatial one, so that we have a coefficient $C_0(g_0)^2$ relating $\langle \mathcal{A}_0' \mathcal{A}_0' \rangle$ to $\langle \mathcal{A}_0 \mathcal{A}_0 \rangle$ and a different one, $C_i(g_0)^2$, connecting $\langle \mathcal{A}_i' \mathcal{A}_i' \rangle$ to $\langle \mathcal{A}_i \mathcal{A}_i \rangle$. On the other hand, the values of $C_0(g_0)$ and $C_i(g_0)$ coincide, within the errors, for the symmetric lattices and it is remarkable that the value of C_0 for W60a agrees within the errors with the value of C_i for W60b being the time extension of W60a equal to the spatial extension of W60b.

We are now ready to show why the discrepancy between the values of θ, relevant to control the gauge-fixing algorithm, and the expectation values of $\theta^{'}$, is natural. In fact the definition of θ ($\theta^{'}$) is given by: $\theta = \frac{1}{V \cdot T} \sum_x \theta(x) = \frac{1}{V \cdot T} \sum_x Tr \; [\Delta(x)\Delta^\dagger(x)]$, where: $\Delta(x) = \sum_\mu \; (A(x) - A(x-\hat{\mu}))$ and A ($A^{'}$) is defined as in eq.(1) (eq.(2)) without ag_0 to the denominator. Then in the continuum variables $\theta = \frac{a^4}{\mathcal{V}} \int d^4x \; (\partial_\mu A_\mu(x))^2$ where $\mathcal{V}$ is the 4-volume in physical units (analogously for $\theta^{'}$). Therefore, while θ vanishes configuration by configuration, as a consequence of the gauge fixing, $\theta^{'}$ is proportional to $(\partial_\mu A_\mu')^2$, which has the vacuum quantum numbers and mixes with the identity. The expectation value of $(\partial_\mu A_\mu')^2$, therefore, diverges as $\frac{1}{a^4}$ so that $\theta^{'}$ will stay finite, as $a \to 0$.

We believe that this discussion on the definition of gauge field operators has a general validity and will survive a more thorough treatment of the gauge-fixing problem.

	W58	W60a	W60b	W64
β	5.8	6.0	6.0	6.4
# Confs	20	100	50	30
Volume	$6^3 \times 6$	$8^3 \times 8$	$8^3 \times 16$	$8^3 \times 16$
$C_i(g_0)$	0.689(3)	0.729(1)	0.729(2)	0.757(2)
$C_0(g_0)$	0.690(7)	0.729(1)	0.750(1)	0.784(2)
a^{-1}	1.333(6)	1.94(5)	1.94(5)	3.62(4)

Table 1
Summary of the lattice parameters used and relative values of C_0 and C_i.

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